Dynamical Trap Effect in Virtual Stick Balancing

Arkady Zgonnikov, Ihor Lubashevsky, Maxim Mozgovoy
University of Aizu, Japan

March 30, 2012

1 Introduction

Wide variety of physical formalism and notions have been used recently in describing social systems and behavior of human as a part of such systems (e.g., see [1, 2]). Particularly, the basic concepts of Newtonian mechanics are commonly applied in the modeling of traffic flow and motion of groups of animals (fish schools, bird flocks, etc.)[3]. The notions of master equation and Hamiltonian as an energy function were used in theory of opinion dynamics and the dynamics of culture and language (e.g.,[4]). Among other concepts of physics that are widely used in social systems analysis are fluid dynamics, Ginsburg-Landau equations and reaction-diffusion systems (e.g.,[5]).

Despite aforementioned advances, one can still note that the mathematical theory of human behavior in social systems is far from being developed well. Apparently, inanimate objects under consideration of Newtonian physics differ substantially in its nature from animate beings, since such features as motivation, morale, memory, learning, etc. are inapplicable to the former. So we may assume that the corresponding mathematical formalism still should be developed in the domain of social systems in addition to the existing notions derived from physical ones.

One of such notions widely met throughout probably all branches of physics is a fixed-point attractor, or stable equilibrium point; it is also commonly used in social psychology [6] as like as the notions of periodic attractor and latent attractor. Nevertheless, social objects and systems in the real world demonstrate anomalous dynamics and irregular behavior which often cannot be reduced to established patterns like equilibrium points or limit cycles. The development of individual, specialized notions which will take into account the pecularities of human being may enable us to better describe and understand these complex systems.

Let us consider an imaginary dynamical system controlled by the operator whose purpose is to stabilize the system near an equlibrium point. We assume that the operator does not react on small variations from this equlibrium, though these variations are clearly recognized by her perception. When the operator is comfortable with the deviation that is lower than certain threshold, she prefers not to intervene system dynamics until the deviation becomes large enough. In other words, any point from certain neighborhood of the equilibrium one is treated equally by the operator. This assumption is in fact due to the phenomenon of human fuzzy rationality [7]. In the present paper we discuss the notion of dynamical trap which was previously introduced in order to mimic this property of the bounded capacity of human cognition [8].

Dynamical traps

In order to illustrate the dynamical trap concept let us assume that considered imaginary dynamical system is described by the following equations

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \Omega(x,y)F(x,y) + \xi(t), \end{aligned} \tag{1}$$

and has an equilibrium in the origin of coordinates (0,0). Here $\Omega(x,y)$ stands for the dynamical trap effect, $F(x,y)$ is regular force and $\xi(t)$ is random factor. $\Omega(x,y)$ could be defined

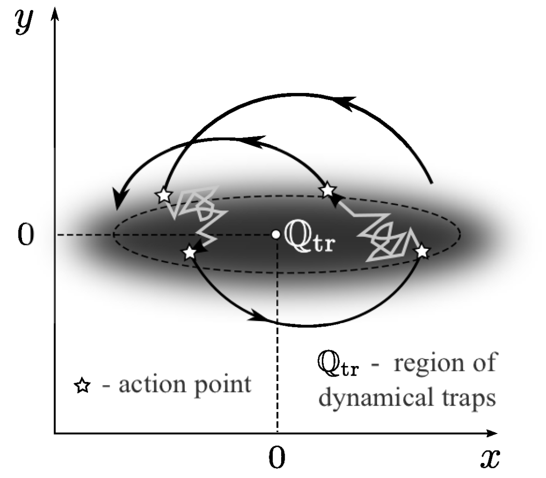

Figure 1: The structure of phase space of system (1)

as follows

$$\Omega(x,y) \approx 0 \text{ if } (x,y) \in \mathbb{Q}_{\rm tr},$$
$$\Omega(x,y) = 1 \quad \text{otherwise},$$

where $\mathbb{Q}_{\rm tr}$ is certain vicinity of the equilibrium point.

In order to explain the meaning of cofactor $\Omega(x,y)$ we consider the behavior of the operator who is approaching desired phase space position $(x=0, y=0)$. Let us assume that if the current position is far from the origin, the operator perfectly follows the optimal control strategy. If the current position is recognized by the operator as "good enough" ($(x,y) \in \mathbb{Q}_{tr}$) (though it may not be strictly optimal) due to her fuzzy rationality, she halts active control over the system so that system dynamics is stagnated in certain vicinity of the desired position. Therefore, $\mathbb{Q}_{tr}$ is called the area of dynamical trap. The structure of the phase space of the described system is presented on Fig. 1.

The investigation of the dynamical traps model was originally inspired by a class of intrinsic cooperative phenomena found in the dynamics of vehicle ensembles on highways [9]; later it was shown that the dynamical trap effect could cause emergent phenomena in the chain of oscillators mimicking the interaction of motivated objects [8, 10]. Among other results obtained in [10], it was demonstrated numerically that the "motivated" oscillator from the particle chain under the presence of dynamical trap forms the specific phase space trajectories (Fig.1). The phase variables distributions were shown to take non-Gaussian forms. The reviewed results demonstrate that the dynamical trap effect could be responsible for establishing of complex patterns of the system motion near the equilibrium point.

Inspite of these achievments up to now there were no experimental evidences of the dynamical trap effect existence in the real world. The purpose of current work is to provide an experimental background to the theoretical framework developed earlier by comparing the results of previous studies on dynamical trap effect and the results of the series of experiments aimed at elucidation of some characteristics of human fuzzy rationality in stick balancing task.

In order to exemplify theoretical studies on the dynamical trap effect we consider the process of the inverted pendulum balancing by human. The task of dynamic stick balancing has been investigated widely from various perspectives; studies on both real-world and virtual experiments are available (see, e.g., [11, 12]). However, attention is mainly paid to the in-depth understanding of the mechanical, physiological and psychological aspects of the process, while we aim to provide an experimental background to the simple model of human behavior which may be useful in modeling complex systems where human decisions play crucial role.

2 Virtual experiments

In present work we focus on the computer-based simulation of two-dimensional inverted pendulum motion in viscous environment. Real-world motion capture-based stick balancing

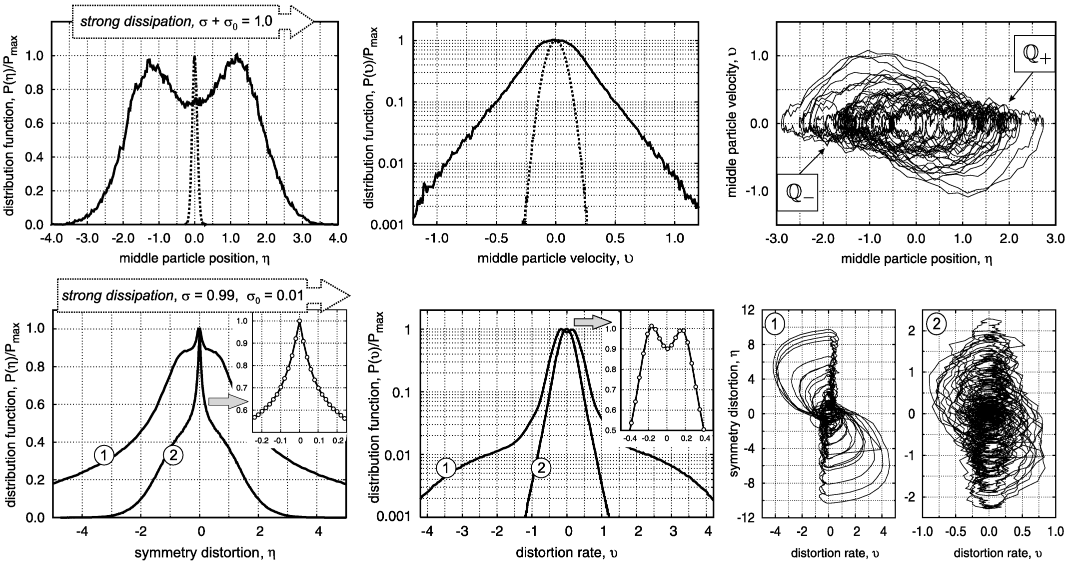

Figure 2: Phase variables densities functions and phase trajectories of a single particle from the ensemble of interacting motivated particles governed by fuzzy rationality [10]. The top three diagrams correspond to the case of the single moving particle oscillating between two fixed neighbors. On the densities diagrams solid lines correspond to the case of strong dynamical trap effect and dotted lines match the absence of dynamical trap. Four bottom diagrams depict phase portraits and densities functions of the particle in the middle of 1000-particle chain with low (label 1) and high (label 2) density of particles.

experiments were also held, as well as virtual experiments simulating stick balancing in non-viscous environment. Preliminary analysis of the experimental data demonstrated that the corresponding dynamical systems exhibit more complex behavior than the system currently under consideration due to the increased number of phase space variables. Therefore its detailed analysis requires an individual consideration and does not fall under the scope of current work.

The mechanical system under consideration is described by the following dimensionless mathematical model:

$$\tau\dot{\theta} = sin\theta - Av(t)cos\theta. \tag{2}$$

Here phase space variable θ is angle between stick and vertical axis, τ is a time scale parameter characterizing the operator perception and the right-hand part of the equation represents the sum of friction and gravity force moments. $v(t)$ stands for the speed of platform motion which is actually the control parameter affected by system operator while A is constant amplifying coefficient of control effort.

It is notable that the phase space of system (2) should comprise not only angle θ but also its derivative $\dot{\theta}$. This assumption is due to the fact that the operator controlling the system evidently perceive the angular velocity of the stick and regulates the value of control effort $v(t)$ based on current values of both factors. Hence, the system dynamics is determined not only by stick angle but by angular velocity as well. The similar approach of phase space extension was previously proposed in the studies on the car following theory [9] where "position-speed" phase space was extended by acceleration as the third independent phase variable.

We developed a simple tool that implements the model described above. The operator has to maintain the angle between the virtual stick and the vertical axis near unstable equilibrium position $\theta_{eq} = 0$ by moving the platform via computer mouse. The total number of subjects participating in the experiment was 12, including both male and female students and professors of different nationalities. Therefore, we achieved participants diversity in nationality, gender and age in order to make the experimental group more-less representative.

A few sessions of the experiments were held. During each session subjects had to control virtual inverted pendulum for the time period of 5 to 20 minutes after 5-minutes adaptation period. To prevent the fatigue effect, sessions were held on the different days. For each

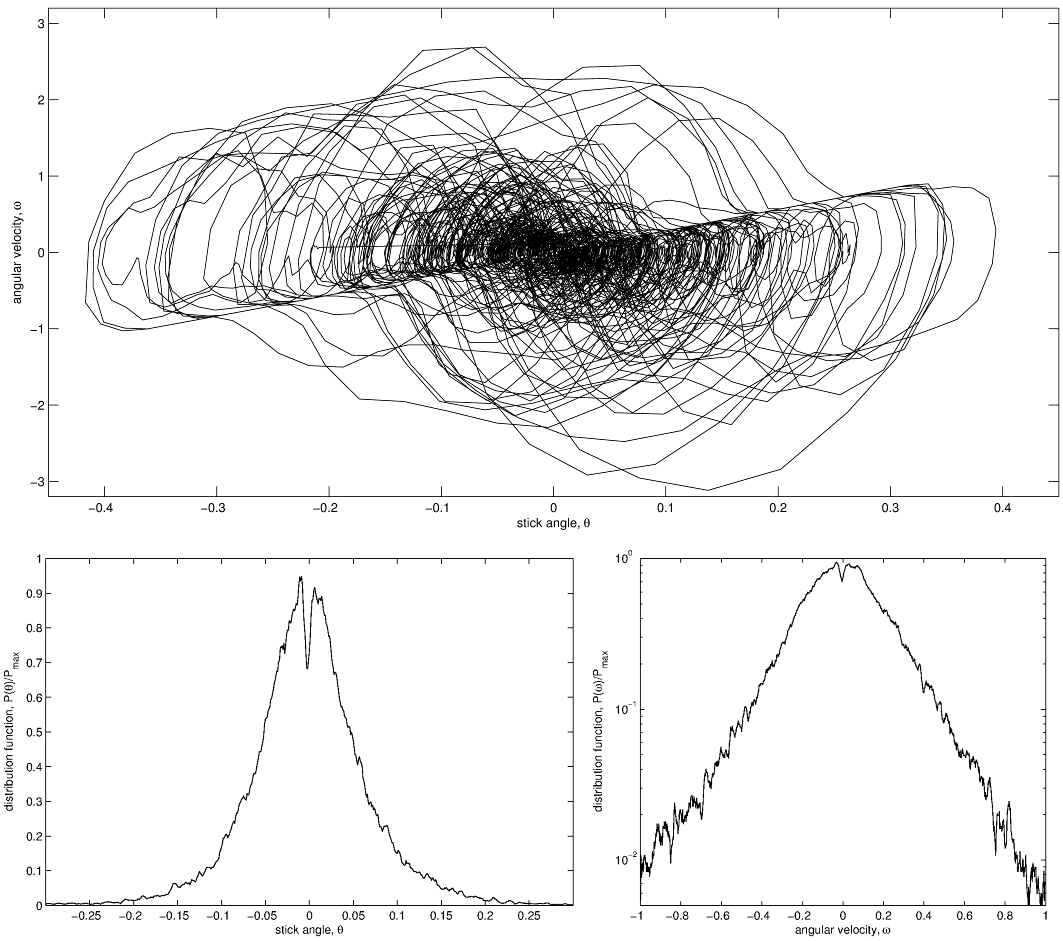

Figure 3: Phase portrait and phase variables distribution (normalized) of the virtual inverted pendulum motion in viscous environment under control of male student during time interval of 5 minutes, $\tau = 0.3$, $A = 0.5$. The distribution of the angular velocity is depicted in logarithmic scale.

participant we have acquired at least three sets of data for various durations of control process.

The numerical data captured from each subject was analyzed separately. It was found that after a short period of adaptation each participant mainly starts to follow the simple strategy of system control:

- wait until the angle or angular velocity of the stick exceeds certain threshold;
- correct the platform postion so that the angular velocity is damped and the stick position is approximately vertical and so on.

From each raw data set obtained we extracted the data required to visualize the phase space trajectories of inverted pendulum motion in "angle—angular velocity" phase space. It was discovered that the phase portraits of the system under human control are extremely similar in their basic properties for all participants and for any considered process duration. It is notable that though the average magnitude of deviation from the equilibrium and average time of continuous balancing vary from one subject to another, the structure of phase portrait is stable within the whole group, as like as the probability distribution functions of phase space variables θ and $\omega = \dot{\theta}$. Fig.2 represents the typical system motion trajectory and corresponding distribution functions.

3 Discussion

Surprisingly, the structure of phase trajectory obtained by the virtual stick under human control is quite similar to the ones of a single oscillator from the particle ensemble studied in [10] (see Fig.1). The certain dissimilarity of the trajectories in the neighborhood of equilibrium points is probably due to the fact that these equilibria are of different nature; the oscillators in the chain are artificially exposed to the external white noise disturbance, while the virtual stick is itself unstable at $\theta = 0$.

This comparison may lead us to the assumption that human behavior during the process of two-dimensional inverted pendulum balancing is in some sense analogous to the behavior of the particle balancing its position between two neighbors described by the dynamical trap model. Furthermore, analysis of system (2) phase variables distributions revealed that probability density functions for both variables have anomalous bimodal form (Fig. 2), as like as corresponding functions (Fig.1) found during the analysis of system of interacting oscillators in [10]. Besides, the form of angular velocity distribution found is like cusp $\propto \exp(-|\omega|)$, which again is anomalous and highly analogous to the results of previous studies on dynamical trap effect. All these facts could be considered as the first experimental evidence of the dynamical trap existence in the real world.

We may therefore expect that bounded capacity of human behavior could be described by the proposed dynamical trap model, which in turn could be used for modeling of wide variety of complex systems comprising large numbers of interacting human beings. Moreover, one may even speculate that the results obtained give evidence to the fact that the standard notion of fixed-point attractor may not be applicable in dynamical systems where human role is crucial due to the phenomena of fuzzy rationality.

Acknowledgements

The work was supported by the Fukushima Prefectural Foundation, Project F-23-1, FY2011.

References

[1] Chakrabarti, B. K., Chakraborti, A., and Chatterjee, A. (eds.) *Econophysics and Sociophysics: Trends and Perspectives*, Wiley VCH Verlag, Weinheim, 2006.

[2] Nowak, A., Strawinska, U. Applications of Physics and Mathematics to Social Science, Introduction to, in *Encyclopedia of Complexity and Systems Science*, ed. Meyers, R. A. SpringerScience+Business Media, LLC., New York, 2009, pp. 322–326.

[3] Castellano, C., Fortunato, S., and Loreto, V. Statistical physics of social dynamics, *Rev. Mod. Phys.* **81**, 2009, pp. 591–646.

[4] Slanina, F. Social Processes, Physical Models of, in *Encyclopedia of Complexity and Systems Science*, ed. Meyers, R. A. SpringerScience+Business Media, LLC., New York, 2009, pp. 8379–8405.

[5] Helbing, D. Traffic and related self-driven many-particle systems, *Rev. Mod. Phys.* **73**, 2001. pp. 1067–1141.

[6] Vallacher, R.R. Applications of Complexity to Social Psychology, in *Encyclopedia of Complexity and Systems Science*, ed. Meyers, R. A. SpringerScience+Business Media, LLC., New York, 2009, pp. 8420–8435.

[7] Dompere, K.K. Fuzzy Rationality. Springer-Verlag, Berlin, 2009.

[8] Lubashevsky, I. Dynamical traps caused by fuzzy rationality as a new emergence mechanism, *Advances in Complex Systems*, in press, 2012.

[9] Lubashevsky, I.A., Wagner, P., Mahnke, R. Rational-driver approximation in car-following theory, *Phys. Rev. E*, 68, 056109, 2003.

[10] Lubashevsky, I.A., Mahnke, R., Hajimahmoodzadeh, M., and Katsnelson, A. Long-lived states of oscillator chains with dynamical traps, *Eur. Phys. J. B*, 44, 63–70, 2005.

[11] Cabrera, J.L., Milton, J.G. On-Off Intermittency in a Human Balancing Task, *Phys. Rev. Lett.*, 89, 158702, 2002.

[12] Suzuki, S., Harashima, F., Furuta, K. Human Control Law and Brain Activity of Voluntary Motion by Utilizing a Balancing Task with an Inverted Pendulum, *Adv. in Hum.-Comp. Inter.*, 2010, 215825, 2010.