Computer Simulation of Stick Balancing. Action Point Analysis

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ABSTRACT

We analyze data collected during the series of experiments aimed at elucidation of basic properties of human perception, namely, the limited capacity of ordering events, actions, etc. according to their preference. Previously it was shown that in a wide class of human-controlled systems small deviations from the equilibrium position do not cause any actions of the system's operator, so any point in a certain neighborhood of equilibrium position is treated as an equilibrium one. This phenomenon can be described by the notion of dynamical traps that was introduced to denote a region in the system phase space where the object under consideration cannot clearly determine the most preferable of the positions that are similar in some sense. According to this concept, the motion of the system in the dynamical trap region is mainly not affected by the operator. The moments of time when the system leaves the dynamical trap region, or in other words, when the operator decides to start or stop the control over the system, are called action points [1]. These moments are seem to be determined intuitively by the operator, and the purpose of our work is to understand the nature of such intuitive decision making process by investigating the action points data obtained from the experiments.

Categories and Subject Descriptors

H.1.2 [**User/Machine Systems**]: Human information processing

Keywords

Human behavior, dynamical systems, action points.

1. INTRODUCTION

During the last decades there has been notable progress in describing social systems and human behavior based on physical notions and mathematical formalism developed in statistical physics and applied mathematics (for a recent review see, e.g., [2, 3, 4, 5, 6]). In particle, the notion of energy functional (Hamiltonian) and the corresponding master equation were employed to simulate opinion dynamics, the dynamics of culture and languages (e.g., [4, 6]); the social force model inheriting the basic concepts from Newtonian mechanics was used to simulate traffic flow, pedestrian motion, the motion of bird flocks, fish school, swarms of social insects (e.g., [4, 7]).

Despite these achievements we have to note that the mathematical theory of social systems and systems desribing human behavior is currently at its initial stage of development. Indeed, animate beings and objects of the inanimate world are highly different in their basic features, in particular, such notions as willingness, learning, prediction, motives for action, moral norms, personal and cultural values are just inapplicable to inanimate objects. This enables us to pose a question as to what *individual* physical notions and mathematical formalism should be developed to describe social systems and human behavior in addition to the available ones inherited from modern physics.

2. DYNAMICAL TRAPS

One of specialized notions used for modeling properties of human cognition is dynamical trap [8, 9]. Here we will refer to dynamical trap as a region in phase space of dynamical system where the system state is treated as an equilibrium one by the system operator due to his imperfect (or fuzzy) rationality. For example, in driving a car the control over the relative velocity is of prime importance for the driver in comparison with the correction of the headway distance to the car ahead. So under normal conditions a driver should eliminate the relative velocity between his car and a car ahead and only then optimize the headway. But according to the concept of dynamical trap the driver recognizes any value of headway distance from a certain range as an optimal one and mostly prefers to slow down his activity not to interrupt the system dynamics after optimizing the relative velocity.

It should be pointed out that a similar notion of dynamical traps was also introduced for relaxation oscillations in systems with singular kinetic coefficients [10] and congested traffic flow [11]. Besides, the concept of dynamical traps is met in describing Hamiltonian systems with complex dynamics (for a review see, e.g., [12]) that denote some re-

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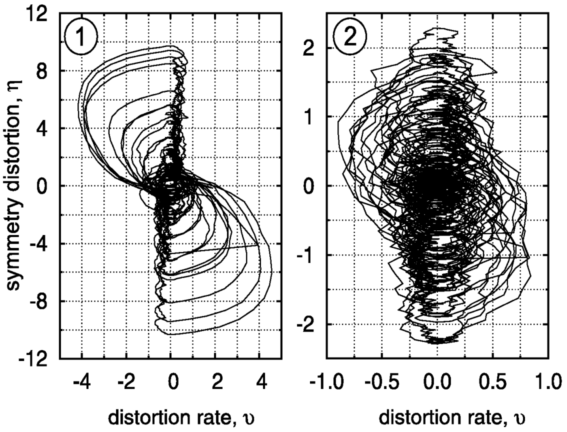

Figure 1: Phase portrait of the multi-particle system motion with low (label 1) and high (label 2) density of particles [8]

gions in the corresponding phase space with anomalously long residence time, however, the nature of the latter traps is different.

In [8] the model of an oscillator chain with dynamical traps was considered. Amongst other results obtained, it was shown that one-dimensional ensemble of particles with motivation exhibits quite complex behavior. Particularly it was demonstrated numerically that individual particle from ensemble forms the specific phase space trajectory (Fig.1). Later we will shown that quite similar behavior can be demonstrated by completely different system which is under our consideration in this paper.

3. EXPERIMENTS

In present work we introduce a series of virtual experiments on balancing the inverted pendulum in viscous environment. Our aim is to analyze based on the results of these experiments the principles of human behavior and decision making during such processes and to understand whether it could be described using the notion of dynamical trap or not.

We consider well-known problem of balancing the inverted pendulum near the vertical position (Fig.2). In this paper we focus on computer-based simulation of pendulum motion in viscous environment (the classical frictionless model, as well as real-world stick balancing process will be considered in future work). The mechanical system under consideration is described by the following dimensionless mathematical model:

$$\tau\dot{\theta} = -cos\theta + A \cdot \upsilon(t) \cdot sin\theta. \qquad (1)$$

Here τ is a time scale characterizing the operator perception and the right-hand part of the equation represents the sum of friction and gravity force moments. $\upsilon(t)$ stands for the speed of cart motion which is actually the control parameter affected by system operator while A is the amplifying coefficient of control effort.

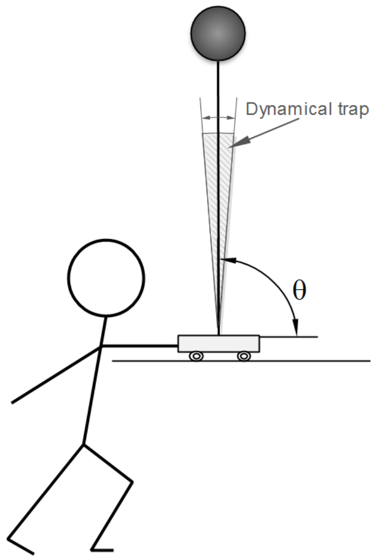

Figure 2: Scheme of the inverted pendulum balancing process

We developed a simple tool that implements the model desribed above. The operator has to maintain the angle between the stick and the horizontal axis near the equilibrium position $\theta_{eq} = \frac{\pi}{2}$ by moving the platform via computer mouse or WiiMote controller, which is closer to the real-life balancing. The experiment was conducted both ways by the group of University of Aizu students.

In order to eliminate the influence of students individual peculiarities on the results of the research we thoroughly checked that there are no significant differences in characteristics of data obtained from each student. Though some artifacts can still be found in the data, the results seem to be stable and are checked to be reproducible in following experiments.

During experiments it was noticed that after certain period of adaptation each student started to follow the simple strategy of system control:

- wait until the angle or angular speed of the stick exceeds certain limit;
- correct the cart postion so that the angular speed is damped and the stick position is approximately vertical and so on.

As a part of analysis of action points, or moments of time when students decided to start/stop the correction of cart position, we have visualized the phase trajectories of the system over the phase variables θ and ω — angle and angular speed of the stick respectively. Over the time span of approximately 2 minutes the system forms the phase trajectory depicted on Fig.3. The parameters values used for the simulation are $\tau = 0.3$ and $A = 0.7$.

It can be easily seen that the phase portrait of system (1) under human control is very similar to the one of a single particle in multi-particle ensemble with dynamical trap in

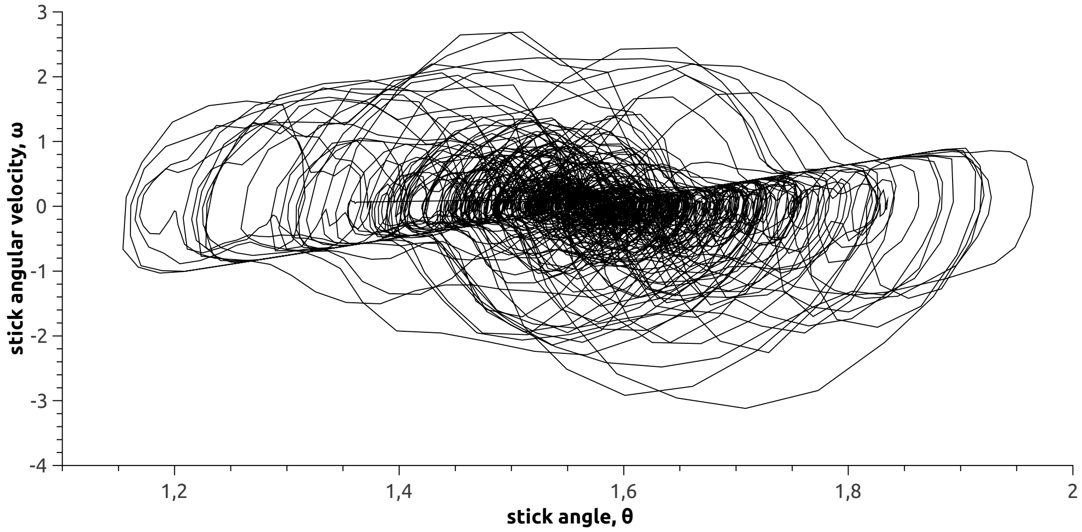

Figure 3: Phase portrait of inverted pendulum motion under human control

case of high particle density (Fig.1). This fact directly leads us to the assumption that the dynamical trap effect actually exists in the process under consideration. Therefore, human behavior during the process of controlling the inverted pendulum in viscous environment may be in principle described in terms of dynamical traps.

4. CONCLUSIONS

A simple model of vertical stick balancing was implemented in order to elucidate some properties of human perception via a series of virtual experiments. By analyzing the sets of action points data collected during the experiments it has been discovered that the behavior of the human-controlled pendulum in viscous environment is very similar to the behavior of previously studied imaginary model of multi-particle system with dynamical trap and therefore could be described by the notion of dynamical traps. In fact, this result may be considered as the first experimental evidence of presence of the dynamical trap effect in human behavior.

5. ACKNOWLEDGMENTS

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